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# Implementation of Time Error Analysis for different Earth Orbits in GPS Satellite

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**Abstract**—*Satellite plays a major role in day to day life such as navigation, telecommunication, military communication, weather and climate and space station etc. The satellite are implemented with GPS system with in it but the small time delay in the clock of the satellite can lead to tremendous loss in communication. This time dilation is mainly due to the General theory and Special theory of relativity. This paper works on the computation and time analysis in GPS satellite for different earth orbits i.e. Low earth orbit, middle earth orbit and high earth orbit. Albert Einstein postulated General and special theory of relativity in 1905 and 1915 which explained the concept of time delay. The computation and error analysis has been done by simulations which are performed in MATLAB software. This paper mainly emphasize on GPS satellite and atomic clock due to the betterment in navigation and communication system in the present scenario. The algorithm has been made and simulated in MATLAB software with the plotting of various graphs such as comparing the time dilation of various earth orbits, synchronization in time dilation.*

## 1. INTRODUCTION

In our day to day life, Global Positioning System (GPS) plays a major role by giving accurate navigational coordinates along with time. We use GPS to locate places using navigational tools. Calculation of exact and precise time value is the most important and fascinating feature of GPS satellite due to which exact location of the GPS device is calculated. GPS satellite uses atomic clocks to measure the onboard time and these values when received on earth are compared with the values computed by the earth's atomic clock[1]. A small error in time value can lead to misguiding or getting wrong coordinate values. Time error correction is most important for GPS as well as other satellites orbiting around earth. There are many ways to calculate the error in values of time including Sagnac effect, Doppler's effect, Relativistic principles, etc. The work in this paper focuses only on the relativistic effects experienced by the satellite in various orbits such as Lower earth orbit (LOE), Middle Earth Orbit (MEO) and Higher Earth Orbit (HEO) and calculates the time error generated by the velocity of the moving satellite (Special relativity theory) and the change of gravitational acceleration (General relativity theory). According to Albert Einstein's 'Special Theory of Relativity 1905' (SRT) and 'General Theory of Relativity 1915' (GRT) there is a time dilation experienced by a moving and a body under influence of gravity respectively. MATLAB software is used to write c/a and p code for Simulink feature to show how time dilation factor vary with increase in altitude above the ground including values for LEO, MEO and HEO. By varying the altitude above sea level, a curve is plotted between the time dilation value and the altitude. GPS satellites move around earth in MEO 20,000 Km above the sea level[3]. They experiences loss of gravitational acceleration and increase in satellite's velocity. The calculated value for this altitude is implemented in GPS satellites and a corresponding time correction is made by changing the ticking rate of the atomic clock onboard. Special relativity concept is explained in **I** using Lorentz transformation. General relativity is explained in **II** using Schwarzschild postulates. Plotting of the time dilation due to special relativity theory and general relativity theory v/s the altitude is explained in **III** using MATLAB algorithm generated using the formulations from above sections. Finally the implementation of time dilation in GPS satellite is showed in **IV** using the calculated values. **V** is the conclusion of the paper explaining how we can implement these errors in GPS satellite along with other parameters.

## 2. RELATIVISTIC EFFECTS IN GPS SATELLITE

There are three relativistic effects which are encountered for GPS i.e. gravitational frequency shift, time dilation and eccentricity effects. Now relativity has been broadly classified into two broad categories General Theory of Relativity and Special Theory of Relativity [5]. The theory of relativity tells us that whenever there is constant movement and change in height with respect to Earth centered, non-rotating inertial reference frame ,the satellite clock speed is affected every time.

### 2.1 Special Theory of Relativity

Special Relativity predicts the variation in the frequency of moving atomic clocks with respect to the stationary atomic clock .It tells us that the atomic clock moving with the GPS orbital speed ticks more slowly than the stationary atomic clock.

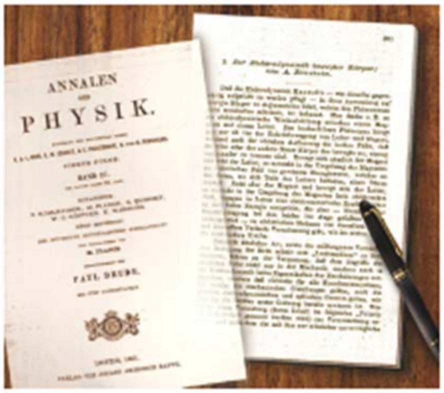


**Fig. 1: The 1905 Paper on Special Relativity**

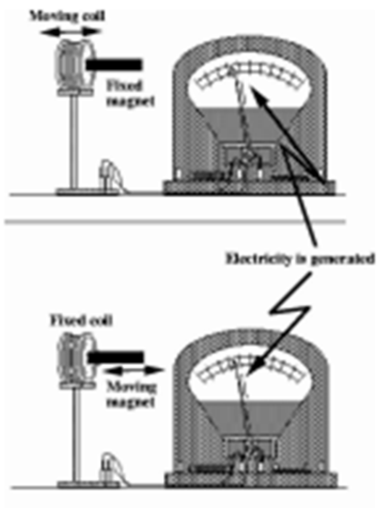



**Fig. 2: Experimental Observation that lead Einstein to the Principle of Relativity**

The figure1 above tells us about the first research paper given by Einstein on Theory of Relativity. The latter figure led to the foundation of Principle of Relativity. According to this, whenver magnet is rotated inside the coil,the current is generated and when the magnet is kept stationary ,coil is rotated the current is generated. So the relative motion between the two objects led Einstein to postulate the Principle of relativity.[7]

Special relativity uses Lorentz transformation for the estimation and calculation of time dilation.

### 2.1.1 Lorentz Transformation

It is the relation between the two reference frames one in which the event has actually taken place and other one in which we want to find the values with respect to the inertial reference frame. Let there are two cordinate planes denotd by K and K', in which K denotes the cordinate palnes in which the event has occurred and K' is any arbitrary reference frame[8]. An event in which it is fixed with respect to space is K with x,y,z cordinates planes with respect to time t. In relatve to K', the same event will be fixed with respct of space with the modified values of cordinate planes x',y',z' with the new time interval t'. For the relative coordiante system the below figure resuts in the relation in terms of equations written below.

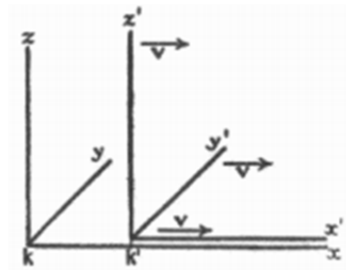



**Fig. 3: Relative Reference Cordinate Frames**

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad (1.1)$$

y' = y (1.2)

z' = z (1.3)

$$t' = \frac{t - \frac{v.x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (1.4)$$

$$\Delta t = -\frac{v^2}{2c^2} \quad (1.5)$$

The velocity values are predicted from orbital formulae's resulting in the estimation of the time dilation for different orbits[8-10]. The above system of equations is known as Lorentz transformation which helps to depict the time dilation in two different frames.

## 2.2 General Theory of Relativity

General Theory is the relativistic theory of gravitation in which Einstein defined gravity as a curvature of space time instead of force. General Relativity is often summarized by the very famous quote of John Wheeler:

"*Space-time tells matter how to move,*
*And matter tells space-time how to curve."*

General Relativity predicts the gravitational frequency shift which tells that the clock near the heavy object moves slower than the clock farther away. Due to the General Theory, GPS clocks move much faster than the Earth clocks. The Schwarzschild formulation is the solution for general theory time dilation. It is the solution which is used to apply space-time in the vicinity of the non-rotating heavy objects[11,13]. The Schwarzschild equations are as follows:

$$t = \frac{t_{\circ}}{\sqrt{1-\frac{2gR}{c^2}}} \quad (2.1)$$

$$\Delta t = \frac{GM_e}{R_e c^2} - \frac{GM_e}{R_{gps} c^2} \quad (2.2)$$

$$\boldsymbol{\Delta t = \frac{1}{c^2}\left(g_e R_e - g_{gps} R_{gps}\right)} \quad (2.3)$$

Where Δt is the time dilation factor.

**2.2.2 Fabric of Space**

In general theory of relativity, Space is assumed as a fabric which curves in the presence of high energy. The heavy bodies like planets of our solar system bends the fabric around them like sun bends the huge amount of fabric of space around it and turn the flat fabric into cone like structure due to which other planet start rotating around it which is origin of centrifugal force.

Time-like, light-like and space-like intervals are three types of notations in special relativity which can be used to classify one dimension curves through curved space-time.

In theory of gravity, you can't really separate the structure of space and time from the particles which is associated with the force of gravity. Therefore the notion of the string is inseparable in the space time in which it moves.

## 3. EARTH ORBITS

Below the altitude of 160 Km, satellite experiences a rapid orbital delay and altitude loss. Therefore all the objects launched are put into orbits at altitude above 160 Km. Lower Earth Orbit (LEO) starts form 160 Km above earth and ends up to 2,000 Km. A satellite should maintain a velocity of 7.8 Km/sec to be in a stable LEO. This velocity increases with altitude. Above the altitude of 2,000 Km, the region for Medium Earth Orbit starts and end till the Geosynchronous Orbit 35,786 Km. All the Navigational, Communicational satellites are put in this region. The most common used altitude is 20,200 Km which yields time period of approx. 12 hours. GPS satellite is put at this altitude so as to move twice around earth. High Earth Orbit (HEO) is the orbits having altitudes above the altitude of 35,786 Km. these orbits have time period greater than 24 hours[12-14]. Any satellite which moves around earth moves according to orbital formulas which are deduced using Newton's second law stating force on a body equals its mass times acceleration by solving two body problem. The distance from center of earth 'r' is given by

$$r = \frac{h^2}{\mu(1+e\cos\emptyset)} \quad (3.1)$$

The velocity $V_p$ (perpendicular) and $V_r$ (radial) at a particular value of 'r' is given by

$$V_p = \frac{\mu(1+e\cos\emptyset)}{h} \quad (3.2)$$

$$V_r = \frac{\mu\, e \sin\emptyset}{h} \quad (3.3)$$

The work done in this paper mainly focuses circular orbits for which e=0. For circular orbits 'r' is given by

$$r = \frac{h^2}{\mu} \quad (3.4)$$

There is only perpendicular velocity which is given by

$$V = \sqrt{\frac{\mu}{r}} \quad (3.5)$$

This velocity formula is put in Lorentz transformation to calculate the time dilated factor for effects from Special Relativity Theory. Time period of the orbit is given by

$$T = \frac{2\pi}{\sqrt{\mu}} \sqrt[3]{r} \quad (3.6)$$

Time period for a GPS satellite is calculated from this formula which comes out to be about 12 hours[15].

## 4. RESULTS AND OBSERVATION

The gravity acceleration (g) varies as we move from the center of the earth and above the surface of the earth. At the surface of earth, g = 9.806650 m/sec$^2$ and varies above the surface as

$$g = g_{\circ}\left(\frac{1}{1+\frac{h}{r}}\right)^2$$

This formula gives the variation of gravitational acceleration with altitude as plotted in MATLAB.

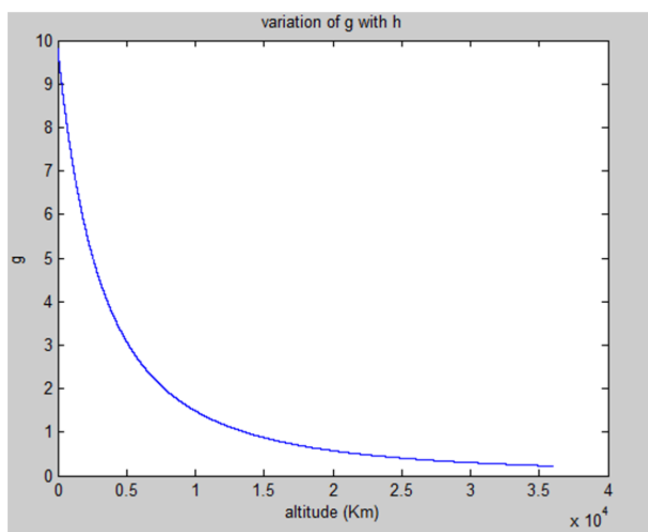



**Fig. 3: Variation of g with Altitude**

The gravitational acceleration variation is put in the formulas for relativistic due to General Relativity Theory [2.1-2.3] to calculate the time dilation factor.

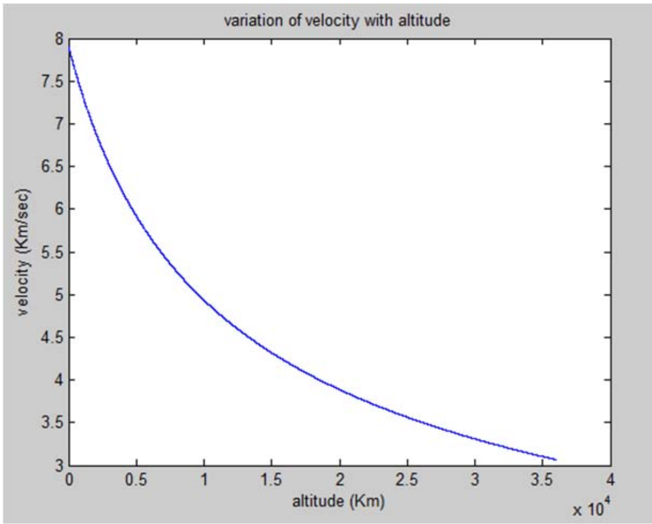



**Fig. 4: Variation of Velocity with Altitude**

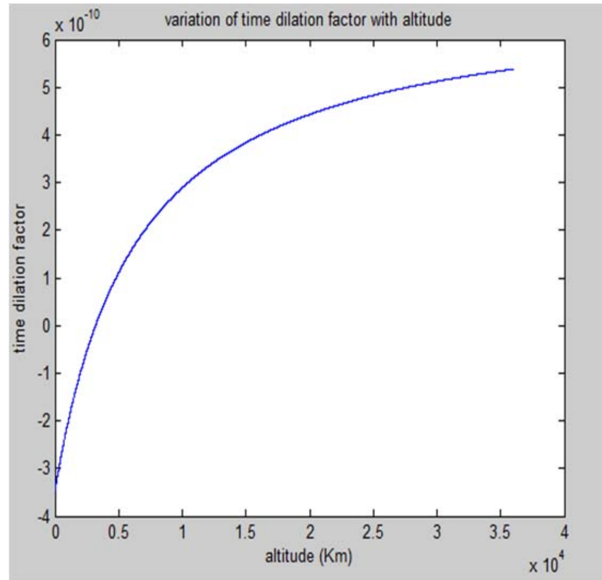
The curve in the graph depicts the variation of orbital velocity with the change in altitude i.e. it tells us that with the increase in the altitude the orbital velocity required to remain in the orbit decreases.

The most important result the time dilation factor variation with the change in altitude .Now here comes two factors change due to general theory of relativity and change due to special theory of relativity. So below table tells the time dilation factor for three different orbits i.e. if any satellite is to be placed in any of the orbits then what will be the time dilation factor that has to be overcome in terms of frequency so that satellite synchronization can be performed effectively with no time lag between earth GPS receivers and GPS satellite.

**Table 1: Time dilation Factor for Different Orbits**

| ORBITS | SPECIAL RELATIVITY (Δt)(*10^-10) | GENERAL RELATIVITY (Δt) (*10^-11) | Δt (*10^-10) |
|---|---|---|---|
| LEO 160 km | -3.3870 | 1.7007 | -3.216 |
| MEO 2000 km | -2.6432 | 16.590 | -0.984 |
| GPS 20200 km | -0.83319 | 52.819 | 4.448 |
| HEO 35786 km | -0.5252 | 58.984 | 5.373 |
| HEO 36000 km | -0.5225 | 59.037 | 5.381 |

The above graph is the variation in the time dilation factor with the change in altitude i.e. for different earth orbits Low Earth Orbit, Middle Earth Orbit and High Earth Orbit .The graph clearly tells that with the increase in altitude the time dilation factor increases extensively. So for high earth orbits, the satellite clock should be designed accordingly in order to have proper navigation and telecommunication by means of GPS satellite.

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